Engineering Notes

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Heliocentric Solar Sail Orbit Transfers with Locally Optimal Control Laws

Malcolm Macdonald*
University of Glasgow, Glasgow,
G12 8QQ Scotland, United Kingdom
Colin R. McInnes†
University of Strathclyde, Glasgow,
G1 1XJ Scotland, United Kingdom
and
Bernd Dachwald‡

DLR, German Aerospace Centre, 51170 Cologne, Germany

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Introduction

OLAR sailing is increasingly being considered by space agencies for future science missions. With the absence of reaction mass from the primary propulsion system arises the potential for new high-energy mission concepts in the mid to far term, such as a Solar Polar Orbiter or an Interstellar Heliopause Probe [1,2]. One of the most time consuming tasks of mission analysis is trajectory generation and optimization. Optimal trajectory generation is a complex field and many schemes exist; however, these are typically characterized as being computationally intensive systems requiring a good degree of engineering judgment [3–6].

The work presented within this Note evolves the use of blended locally optimal control laws from planet-centered solar sail trajectory generation into heliocentric applications [1,7–10]. The primary advantage of locally optimal control laws is the speed with which they can be implemented in a trajectory calculation, giving results up to several orders of magnitude quicker than either direct or indirect optimization methods. The primary disadvantage of locally optimal control laws is the nonoptimal nature of the method and resulting solution, and that the position fixing elements cannot be optimized. The approach adopted previously by the authors is that the weight functions used to blend the control laws should be independent of time, thus the osculating orbit elements set the weight of each control law before blending [7]. In 2003 Petropoulos proposed a similar approach for solar electric propulsion (SEP) orbit transfers [11]. Traditionally the weight of each control law had been set as a function of time, thus negating the time independent advantages of locally optimal control laws.

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In [8] locally optimal control laws were used in the generation of complex, planet-centered solar sail transfers and station-keeping maneuvers. The control method developed in [8], called accessibility and deficit (AⁿD) blending, seeks to give each individual control law a relative importance before defining the final weight functions and thus the blended control vector. The deficit of each element from the final target value is considered. Additionally, the efficiency or accessibility of any attempt to alter an orbital element is considered, thus avoiding inefficient use of the sail such as prolonged periods of high pitch. The deficit and accessibility scores of each element are added together to gain the AⁿD score, which is then multiplied by a constant to determine the final weight of each element [8]. The use of constants allows the control system to be fine tuned to increase optimality and essentially reduces the trajectory optimization problem from finding the pitch and clock angle control history to finding a small set of constants. The selection of appropriate constants is intuitive, as will be demonstrated, although engineering judgment (or automation) allows a more rapid convergence toward the most favorable solution. Note, in [8] the deficit score is found through application of an ideal solar sail force model. However, within this evolution the deficit score is now found through application of the same nonideal sail force model used to determine the sail thrust vector within the trajectory propagation; as such the assumption of an ideal sail when deriving the control laws no longer compromises the control laws efficiency when a nonideal sail is used for trajectory propagate.

Two heliocentric mission scenarios will be considered, where the results will be compared with similar scenarios analyzed using a sequential quadratic programming (SQP) method, a local optimizer, and an evolutionary neurocontroller, a global optimizer.

Solar Sail Trajectory Model

Modified equinoctial elements are utilized in the equations of motion, which are propagated using an explicit, variable step size Runge–Kutta (4, 5) formula, the Dormand–Price pair, and a single-step method [12–14]. Relative and absolute error tolerances are set at 10^{-12} . Solar electromagnetic energy flux is assumed constant at all times, adopting an accepted mean value of $1368~\rm J\cdot s^{-1}\cdot m^{-2}$ at 1 AU [1]. The sail force model applied can be that of an ideal solar sail or of a nonideal sail, such as the parametric or optical force models [15]. All trajectories assume an Earth departure C_3 of zero, with all planetary positions found from an analytical propagation of the planetary ephemeris at J2000 (corresponding to the Julian date 2451545.0 terrestrial time). The sail control angles are defined in the standard form with respect to a sun-sail line reference frame [1]. Thus, the pitch angle is defined between 0 and 90 deg with the clock angle defined between 0 and 360 deg.

Earth-Mercury Transfer

Mercury is an attractive destination for solar sail missions due to the proximity of the sun which provides high-energy flux levels [16]. Many solar sail missions to Mercury have been proposed, ranging from an orbiter through to a complex sample return mission [1,8,9,17–20]. The generation of an Earth–Mercury transfer trajectory thus allows an excellent case study for heliocentric AⁿD blending, with ample trajectories available within the literature for comparison. Because of the proximity of Mercury to the sun the

^{*}Currently at SciSys Ltd., Real-Time Systems Group; malcolm.macdo-nald@scisys.co.uk.

[†]Professor, Department of Mechanical Engineering; colin.mcinnes@strath.ac.uk. Member AIAA.

[‡]Scientist, currently at DLR, Oberpfaffenhofen, Germany, Mission Operations Section; bernd.dachwald@dlr.de. Member AIAA.

required sail characteristic acceleration is relatively low, however, as a result of this an Earth–Mercury transfer becomes a multirevolution transfer which increases the computational difficulty, especially for traditional optimization techniques. We define sail characteristic acceleration as the acceleration imparted on the solar sail at 1 AU when the sail is trimmed to zero pitch.

Following much of the previous literature a characteristic acceleration of $0.25~{\rm mm\cdot s^{-2}}$ is assumed for the Earth–Mercury transfer, using an ideal sail force model [1–20]. It was found that using AⁿD blending the transfer should be split into two phases, the first concentrating on lowering the semimajor axis, the second on increasing orbit eccentricity and inclination to match those of Mercury. The first phase is 753.3 days in duration, with the second requiring a further 298.3 days. Note that the actual demarcation point between the two phases is typically found through sound judgment and a little trial and error. The constants used on the AⁿD scores are denoted as corresponding element = constant used. Thus, in phase 1 the constants used on the AⁿD scores were a=1.00, e=0.29, and i=0.25 and then $e=0.455, r_p=0.865,$ and i=0.53 during phase 2.

The elements not detailed were multiplied by zero, thus removing them from consideration. Such simplifications can be made to rapidly produce a trajectory solution which is a good approximation of the best-case rendezvous scenario. A plot of the transfer is shown in Fig. 1, where we see the change of the semimajor axis, eccentricity, and inclination. From Fig. 1 it is seen that the orbit inclination remains low until the semimajor axis has been reduced, thus allowing the inclination to be increased more rapidly due to the shortened orbit period. The sail control angles generated by the AⁿD blending algorithms were found to be oscillatory in nature and to correspond well with a 51 node SQP generated Earth–Mercury transfer which has a duration of 1041 days [19], that is to say, 10 days or less than 1% shorter than the transfer generated by AⁿD blending. Note that the SQP result was the best found in the literature and is thus used for reference.

Interstellar Heliopause Probe

Several missions to the heliopause and beyond have previously been studied using many different propulsion systems [21–28]. The optimal propulsion system depends on the technology level assumed and the mission constraints imposed. Recently, the concept of technology reference studies (TRS) has been introduced to focus the development of strategically important technologies of likely relevance to future science missions [29]. Within this Note we will loosely follow the TRS requirements for the Interstellar Heliopause Probe (IHP) mission analysis, where it has been shown that a solar sail is the optimal propulsion system [28]. The TRS states that the

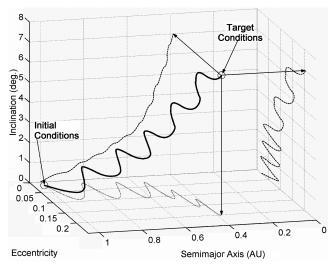


Fig. 1 Diagram of a, e, and i, with the projection of the route into the axis also shown.

spacecraft should reach 200 AU in 25 years. Furthermore, the sail should be jettisoned at 5 AU to eliminate any potential interference caused by the solar sail on the local space environment.

A recent engineering note used the locally optimal energy gain/ reduction control law to generate solar system escape trajectories which, for high-performance sails, were relatively close to those produced using complex optimization procedures [30]. This work, however, struggled to produce near-optimal escape trajectories when the sail performance was lowered and the trajectory became more complex. We also note that for a high-performance sail the solar system escape trajectory is relatively simple; hence the adopted method within [30] would be expected to approximate the global optimal solution.

Solar radiation flux drops off as $1/r^2$, thus a solar sail becomes increasingly ineffective at large solar distances. Equally solar sails become increasingly effective at low solar radii. It was thus realized by Sauer that a close approach of the sun by a solar sail would allow sufficient energy to be gained to rapidly escape the solar system, and such a maneuver is often termed a solar photonic assist (SPA) [15]. A thermal limit of 0.25 AU will be adopted for most trajectories within this Note, however, as AⁿD blending enables rapid generation of near-optimal trajectories the impact on trip time of varying the thermal limit will also be presented. Similarly, the effect of a nonideal sail will be analyzed using the optical force model. The term ideal sail is used to describe a sail which is modeled as a perfect reflector; the term nonideal sail is used to describe a sail which is modeled using the optical force model. The optical force coefficients used are those given in [15]. Some work has been conducted previously using this nonideal sail force model for rendezvous trajectories and for heliopause trajectories [17,31–35]. Thereby it was found that flight times are typically 5%, and on occasion up to 10%, longer for the nonideal sail force model.

During an initial analysis it was found that the most favorable 2-D trajectory to 200 AU, with a minimum radius 0.25 AU, using an ideal sail with characteristic acceleration 1.5 mm \cdot s $^{-2}$ has a single aphelion passage of approximately 2.83 AU and reaches 200 AU in 22.73 years. We note that the instantaneous aphelion value at perihelion passage is 10.7 AU. The velocity of the spacecraft at 5 AU, the sail jettison point, for this trajectory is 10.50 AU \cdot yr $^{-1}$.

Using a start epoch of 03 January 2030, an initial analysis of 3-D trajectories suggested that the optimal aphelion passage would increase from the 2-D scenario, to just over 3 AU, with the velocity at 5 AU and trip time remaining very similar to the 2-D case. A start epoch of 03 January (Earth perihelion) gives an azimuth of order 230 deg at 200 AU. We thus estimate the optimal Earth departure date to occur in late January. Using this estimate a launch window analysis was performed for a 1.5 mm \cdot s⁻² ideal sail, with a thermal limit of 0.25 AU. It was found that for a given launch date the spacecraft azimuth at 200 AU could be varied by increasing or decreasing the aphelion passage radius, a larger azimuth being gained by increasing the aphelion passage radius. It was found that the optimal launch date for a 1.5 mm \cdot s⁻² ideal sail, with a thermal limit of 0.25 AU, was 26 January 2030. The trip time to 200 AU is

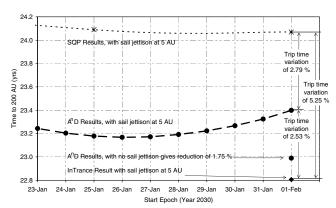


Fig. 2 Launch date scan for an ideal sail with characteristic acceleration 1.5 mm \cdot s⁻², thermal limit of 0.25 AU.

23.17 years, with a spacecraft velocity of $10.47~{\rm AU}\cdot{\rm yr}^{-1}$ at 5 AU. The best open azimuth 2-D trip time found was just over 5 months (<2%) shorter than the 3-D trajectory, with the velocity at 5 AU very similar. The launch date scan is shown in Fig. 2 from 23 January 2030 until 01 February 2030, with a maximum azimuth error at 200 AU of 0.2 deg. Furthermore, it is noted that the elevation convergence is within 10^{-8} deg for all AⁿD blending generated IHP trajectories within this Note.

The constants applied to the A^nD scores for the 26 January trajectory in phase 1 were e=1, a=0.54, and i=0.35602, with the second phase using the locally optimal semimajor axis control law exclusively. The inclination constant is found iteratively, such as to match the inclination to the target value as close to the end of phase 1 as possible. It was found that if the sail was used beyond 5 AU only a small reduction in trip time was gained.

To quantify the optimality of the AⁿD blending method we compare the best-case trajectory with independently generated trajectories using SQP methods (a local optimizer) and an evolutionary neurocontroller, InTrance (a global optimizer) [17]. The same sail characteristic acceleration, thermal limit, and sail force model were used in these methods, with the sail being jettisoned at 5 AU. The optimal duration of the SQP generated trajectory, using 201 control nodes, is shown in Fig. 2, where we see that the optimal launch date was found to be 01 February, giving a trip time of 24.07 years, 2.79% longer than the equivalent AnD trajectory on 01 February. To quantify the optimality of the AⁿD blending method we thus compare against the InTrance results only. To maintain consistency, the InTrance trajectory start epoch was fixed as 01 February 2030, with the azimuth and elevation also constrained, thus allowing the suboptimal nature of both the SQP and AⁿD results to be found. Note the end constraints within the InTrance model are ± 0.1 AU of the target point in space. It is seen in Fig. 2 that the InTrance optimal trip time to 200 AU is 22.81 years, which is 7 months shorter than the equivalent AⁿD trajectory and over 15 months shorter than the equivalent SQP result. We conclude that the AⁿD blending method has generated a trajectory to within 2.5% of the solution found using InTrance, a global optimization method, whereas the SQP method has generated a trajectory which is 5.25% slower than the solution found using InTrance.

We now consider a characteristic acceleration of 1 mm \cdot s⁻² to investigate dual loop trajectories. It was found that a low velocity solar system escape could be achieved with a single SPA with this level of sail performance. Note that it is exactly this scenario which previous attempts to use locally optimal methods to produce solar system escape trajectories have struggled with and typically produced very suboptimal results. Launch was fixed at 03 January 2030 and no attempt was made to constrain the spacecraft azimuth at 200 AU, thus allowing the near-optimal trajectory to be rapidly identified. The radius of the first aphelion passage is minimized by applying the perihelion reduction locally optimal control law. The second phase of the trajectory targets the second aphelion passage radius, much as during the first phase of the single revolution trajectories previously discussed. A third phase is thus used as the energy boost phase during the second perihelion passage, where once again only the semimajor axis controller is used.

Trip time to 200 AU with an ideal and nonideal sail of characteristic acceleration $1 \text{ mm} \cdot \text{s}^{-2}$ is shown in Fig. 3 for a partially constrained 3-D trajectory, that is , open azimuth and 7.5 deg elevation at 200 AU, along with the corresponding velocity at 5 AU for each of the most favorable trajectories found. We note that a nonideal sail typically adds in the region of 5% to the ideal sail trip time, as was found in [35].

Though not shown in Fig. 3 the minimum trip times generated using SQP with an ideal sail, at 0.20 and 0.25 AU thermal limit, were found to be 6 and 3% longer, respectively, than the exactly equivalent A^nD result. Figure 3 shows the near-equivalent results generated using InTrance. We note at this stage that the method of constraint definition within InTrance does not allow for an elevation constraint to be set without an azimuth constraint. Rather, within InTrance the azimuth and elevation are constrained and the launch date remains open, where it was fixed within the A^nD analysis. This distinction is a

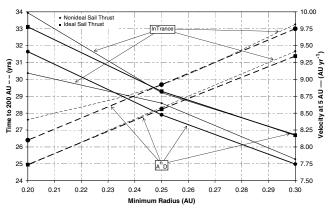


Fig. 3 Trip time to 200 AU (dash) against minimum radius and velocity at 5 AU (solid) for ideal and nonideal sails with characteristic acceleration 1 mm \cdot s⁻².

small but bothersome factor as it means that the InTrance results in Fig. 3 cannot be stated as exactly equivalent, only near equivalent. However, it does allow analysis of the effect of such a simplification. We recall from earlier that the change from a fixed launch date to open launch date resulted in an increase in trip time of approximately 1.3%, using AⁿD. Thus, so long as the InTrance solution is less than 1.5% different from the AⁿD solution we can assume the AⁿD solution to be near optimal. It was found that the trip times in Fig. 3 generated using InTrance are mostly slower than the near-equivalent AⁿD solution; however, all are within 1% except for the nonideal sail scenario with minimum radius 0.20 AU, which is 1.21 years or 4.4% slower. We thus conclude that while the AⁿD results are providing excellent rapid quantification of the trip time to 200 AU, the open azimuth approximation can on occasion be an oversimplification.

Conclusions

A previous (planetocentric) method of assessing the relative importance of orbit elements during solar sail transfers called accessibility and deficit, AⁿD, blending has been evolved, allowing rapid generation of heliocentric trajectories by blending locally optimal control laws.

An Earth–Mercury trajectory was presented to demonstrate the capability of AⁿD blending when attempting to find planet-to-planet transfer trajectories. It was found that the AⁿD generated trajectory duration was within 1% of the sequential quadratic programming, SQP, generated trajectory. Thus, AⁿD blending can provide a very good rapid assessment of such a mission scenario, or provide an excellent initial guess for further optimization as part of a detailed mission analysis.

The AⁿD blending method has also been demonstrated for generation of sail trajectories to 200 AU, where it was repeatedly shown that AⁿD generated trajectories were very similar in duration to the transfer time found using an evolutionary neurocontroller, which is expected to be near global optimal. Furthermore, the AⁿD generated trajectories were consistently more efficient than SQP generated trajectories. AⁿD blending has been shown to be able to optimize a solar sail trajectory using a nonideal sail force model. The AⁿD blending results clearly demonstrate that a nonideal sail will require a characteristic acceleration of approximately 1.5 mm · s⁻² to reach 200 AU in 25 years, assuming no optical surface degradation.

We conclude that AⁿD blending is a highly efficient method for the rapid generation of trajectories, reducing the trajectory optimization problem from finding the pitch and clock angle control history to finding a small set of constants by which to multiply the AⁿD score. Thus, in the Earth–Mercury transfer trajectory where the SQP method had to optimize 102 data points (51 nodes of pitch and 51 nodes of clock) the AⁿD method reduces this to only 6 data points, split evenly over two trajectory phases, a very significant reduction in computational effort. Furthermore, the 200 AU trajectories simplify the equivalent SQP problem from 402 data points to 5 data points for

the dual loop trajectories and only 3 for the single loop trajectories, while also providing trajectories which outperform the SQP solution. AⁿD blending allows swift and accurate mission analysis while also providing an excellent initial guess to other optimization methods.

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References

- McInnes, C. R., "Solar Sailing: Technology, Dynamics and Mission Applications," Springer-Verlag, London, 1999.
- [2] Macdonald, M., Hughes, G. W., McInnes, C. R., Lyngvi, A., Falkner, P., and Atzei, A., "The Solar Polar Orbiter: A Solar Sail Technology Reference Study," *Journal of Spacecraft and Rockets* (submitted for publication).
- [3] Sauer, C. G., "Optimum Solar Sail Interplanetary Trajectories," AIAA Paper AIAA-76-792, Aug. 1976.
- [4] Van der Ha, J. C., "The Attainability of the Heavenly Bodies with the Aid of a Solar Sail," German Society for Aeronautics and Astronautics, DGLR Paper 80-012, March 1980.
- [5] Otten, M., and McInnes, C. R., "Near Minimum-Time Trajectories for Solar Sails," *Journal of Guidance, Control, and Dynamics*, Vol. 24, No. 3, 2001, pp. 632–634.
- [6] Hughes, G., and McInnes, C. R., "Small-Body Encounters Using Solar Sail Propulsion," *Journal of Spacecraft and Rockets*, Vol. 41, No. 1, Jan.–Feb. 2004, pp. 140–150.
- [7] Macdonald, M., and McInnes, C. R., "Realistic Earth Escape Strategies for Solar Sailing," *Journal of Guidance, Control, and Dynamics*, Vol. 28, No. 2, 2005, pp 315–323.
- [8] Macdonald, M., and McInnes, C. R., "Analytical Control Laws for Planet-Centred Solar Sailing," *Journal of Guidance, Control, and Dynamics*, Vol. 28, No. 5, 2005, pp. 1038–1048.
- [9] Macdonald, M., "Analytical Methodologies for Solar Sail Trajectory Design," Ph.D. Dissertation, Department of Aerospace Engineering, University of Glasgow, May 2005.
- [10] Macdonald, M., and McInnes, C. R., "Analytic Control Laws for Near-Optimal Geocentric Solar Sail Transfers (AAS 01-473)," Advances in Astronautical Sciences, Vol. 109, No. 3, 2001, pp. 2393–2413.
- [11] Petropoulos, A. E., "Simple Control Laws for Low-Thrust Orbit Transfers, AAS-03-630," Advances in Astronautical Sciences, Vol. 116, No. 3, 2003, pp. 2031–2049.
- [12] Giacaglia, G. E. O., "The Equations of Motion of an Artificial Satellite in Nonsingular Variables," *Celestial Mechanics*, Vol. 15, 1977, pp. 191–215.
- [13] Walker, M. J. H., Ireland, B., and Owens, J., "A Set of Modified Equinoctial Elements," *Celestial Mechanics*, Vol. 36, 1985, pp. 191– 215
- [14] Dormand, J. R., and Price, P. J., "A Family of Embedded Runge-Kutta Formulae," *Journal of Computing and Applied Mathematics*, Vol. 6, 1980, pp. 19–26.
- [15] Wright, J. L., "Space Sailing," Gordon and Breach Science Publishers, Philadelphia, 1992.
- [16] Macdonald, M., and McInnes, C. R., "Seasonal Efficiencies of Solar Sailing Planetary Orbit," IAC Paper IAC-02-S.6.01, Proceedings of 53rd International Astronautical Congress, 2002.
- [17] Dachwald, B., "Low-Thrust Trajectory Optimisation and Interplanetary Mission Analysis Using Evolutionary Neurocontrol," Ph.D. Thesis, Universität der Bundeswehr München, Fakultät für Luft-und Raumfahrttechnik, Institut für Raumfahrttechnik, April 2004.

- [18] Macdonald, M., and McInnes, C. R., "Solar Sail Capture Trajectories at Mercury," AIAA Paper AIAA-2002-4990, 5–8 Aug. 2002.
- [19] Hughes, G., Macdonald, M., and McInnes C. R., "Analysis of a Solar Sail Mercury Sample Return Mission," IAC Paper IAC-04-Q.2.B.08, Proceedings of 53rd International Astronautical Congress, 2002.
- [20] McInnes, C. R., Hughes, G., and Macdonald, M., "Low Cost Mercury Orbiter and Mercury Sample Return Missions Using Solar Sail Propulsion," *The Aeronautical Journal*, Paper No. 2790, Aug. 2003, pp. 469–478.
- [21] Kluever, C. A., "Heliospheric Boundary Exploration Using Ion Propulsion Spacecraft," *Journal of Spacecraft and Rockets*, Vol. 34, No. 3, May–June 1997, pp. 365–372.
- [22] Leipold, M., Fichtner, H., Heber, B., Groepper, P., Lascar, S., Burger, F., Eiden, M., Niederstadt, T., Sickinger, C., Herbeck, L., Dachwald, B., and Seboldt, W., "Heliopause Explorer: A Sailcraft Mission to the Outer Boundaries of the Solar System," ESA Special Publication SP-542, Sept. 2003.
- [23] Mewaldt, R. A., Kangas, J., Kerridge, S. J., and Neugebauer, M., "A Small Interstellar Probe to the Heliospheric Boundary and Interstellar Space," *Acta Astronautica*, Vol. 35, 1995, pp. 267–276.
- [24] Noble, R. J., "Radioisotope Electric Propulsion of Sciencecraft to the Outer Solar System and Near-Interstellar Space," *Acta Astronautica*, Vol. 44, Nos. 2–4, 1999, pp. 193–199.
- [25] Wallace, R. A., "Precursor Missions to Interstellar Exploration," IEEE Paper 114, March 1999.
- [26] Wallace, R. A., Ayon, J. A., and Sprague, G. A., "Interstellar Probe Mission/System Concept," IEEE Paper No. 53, March 2000.
- [27] Zurbuchen I, T. H., Prashant, P., Gallimore, A., Scheeres, D., Murphy, N., Zank, G., Malhotra, R., Funsten, H., and the NASA Interstellar Probe Vision Mission Team, "Interstellar Probe: Breakthrough Science Enabled by Nuclear Propulsion," *Electronic Proceedings of the 55th International Astronautical Congress*, IAC, Vancouver, Oct. 2004.
- [28] Lyngvi, A., Falkner, P., Kemble, S., Leipold, M., and Peacock, T., "The Interstellar Heliopause Probe," *Electronic Proceedings of the 55th International Astronautical Congress*, IAC, Vancouver, Oct. 2004.
- [29] Lyngvi, A., Flakner, P., Renton, D., v.d.Berg, M. L., and Peacock, A., "Technology Reference Studies", Electronic Proceedings of the 55th International Astronautical Congress, IAC, Vancouver, Oct. 2004.
- [30] Sharma, D. N., and Scheeres, D. J., "Solar-System Escape Trajectories Using Solar Sails," *Journal of Spacecraft and Rockets*, Vol. 41, No. 4, July–Aug. 2004, pp. 684–687.
- [31] Dachwald, B., "Interplanetary Mission Analysis for Non-Perfectly Reflecting Solar Sailcraft Using Evolutionary Neurocontrol," Advances in the Astronautical Sciences and Astrodynamics, Vol. 116, No. 2, 2003, pp. 1247–1262.
- [32] Colasurdo, G., and Casalino, L., "Optimal Control Law for Interplanetary Trajectories with Nonideal Solar Sail," *Journal of Spacecraft and Rockets*, Vol. 40, No. 2, 2003, pp 260–265.
- [33] Dachwald, B., "Minimum Transfer Times for Nonperfectly Reflecting Solar Sailcraft," *Journal of Spacecraft and Rockets*, Vol. 41, No. 4, 2004, pp 693–695.
- [34] Mengali, G., and Quarta, A. A., "Optimal Three-Dimensional Interplanetary Rendezvous Using Non-Ideal Solar Sail," *Journal of Guidance, Control, and Dynamics*, Vol. 28, No. 1, 2005, pp. 173–177.
- [35] Dachwald, B., "Optimal Solar Sail Trajectories for Missions to the Outer Solar System," *Journal of Guidance, Control, and Dynamics*, Vol. 28, No. 6, 2005, pp. 1187–1193.

D. Spencer Associate Editor